Third Semester B.Tech. Degree Examination, April 2015
(2013 Scheme)
13.301 : ENGINEERING MATHEMATICS – II
(ABCEFHMNPRSTU)

Time : 3 Hours Max. Marks : 100

Instructions : *1) Answer **all** questions from Part **A**. **Each** question carries **4** marks.*
*2) Answer **one full** question from each Module of Part **B**. **Each** full question carries **20** marks.*

PART – A

1. Find the angle between the normals to the surface $xy = z^2$ at the points $(-2, -2, 2)$ and $(1, 9, -3)$.
2. Find the workdone by the force $\overline{F} = x\hat{i} + 2y\hat{j}$ in moving a particle from (0, 0) to (2, 2) along the curve $2y = x^2$.
3. Find the Fourier cosine transform of e^{-5x}.
4. Form the partial differential equation by eliminating the arbitrary functions from $z = f(y + 3x) + g(y - 3x) + \frac{x^3 y}{6}$.
5. Solve the equation $U(x, t) = e^{-t} \cos x$ with $U(x, 0) = 0$ and $\frac{\partial u}{\partial t}(0, t) = 0$ by the method of separation of variables.

PART – B

Module – I

6. a) Show that $\overline{F} = (z^2 + 2x + 3y)\hat{i} + (3x + 2y + z)\hat{j} + (y + 2xz)\hat{k}$ is irrotational but not solenoidal.
   b) If $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\overline{r}|$ show that $\nabla^2 r^n = n(n+1) r^{n-2}$.
   c) Use Divergence theorem to evaluate $\iint_S \overline{F}.\hat{n}\,ds$ where $\overline{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
7. a) Find the constants a, b, c so that $\overline{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.
   b) If $\overline{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$. Find $\int_c \overline{F}.d\overline{r}$ where C is the rectangle in the xy plane bounded by $x = 0, x = a, y = 0, y = b$.
   c) Use Green's theorem in the plane to evaluate $\int_C (x^2 - 2xy)\,dx + (x^2 y + 3)\,dy$ where C is the boundary of the region bounded by $y = x^2$ and $y = x$.

P.T.O.

Module – II

8. a) Find the Fourier series of period $2l$ for the function

$$f(x) = l - x, 0 \le x \le l$$
$$= 0, \quad l \le X \le 2l$$

b) Expand $f(x) = \pi x - x^2$ as a half range sine series in the range $(0, \pi)$.

c) Find the Fourier transform of $f(x) = x, |x| < a$

$$= 0, |x| > a, a > 0.$$

9. a) Find the Fourier series of $f(x) = x - 2x^2, -\pi < x < \pi$.

b) Expand $f(x) = x + \pi, 0 \le x \le \pi$

$= -x + \pi \quad -\pi \le x \le 0$, given that f(x) is periodic with period 2π.

c) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$

Module – III

10. a) Solve $\frac{\partial^2 z}{\partial x^2} + a^2 z = 0$, given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = a$.

b) Find the singular integral of $z = px + qy + p^2 - q^2$.

c) Solve : $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$.

11. a) Solve the equation $(D^4 - D'^4)z = e^{x+y}$.

b) Solve : $z = xp^2 + qy$.

c) Solve : $yp = 2xy + \log q$.

Module – IV

12. a) Find the variable separable solution of the heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

b) If a string of length 'l' is initially at rest in the equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = V_0 . \sin^3 \frac{\pi x}{l}, 0 < x < l$. Find the displacement y (r, t).

13. a) A tightly stretched string of length 'l' is fastened at both ends. Motion is started by displacing the string into the form $kx(l - x)$ from which it is released at time $t = 0$. Find the displacement y (x, t).

b) A rod of length 'l' has its ends A and B kept at 0°C and 80°C respectively until slate-state conditions prevail. If B is suddenly reduced to 0°C and kept so, while that of A is maintained, find the temperature function u(x, t).